Optimal Transport and Monge-Ampère equations

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The optimal transport problem, coined by Gaspard Monge in 1781, asks to find the cheapest way of transferring a pile of dirt to an embankment, given the cost of transporting a unit of mass from one location to another. The modern mathematical formulation of this problem leads to interesting theoretical questions in the calculus of variations and nonlinear partial differential equations and allows for applications to a myriad of other disciplines, including but not limited to fluid dynamics, cosmology, matching and pricing in economics, image processing, and machine learning. Due to the ubiquity of optimal transport in various branches of applied mathematics, it is necessary to develop robust numerical methods for the precise approximation of optimal maps. In certain cases, this is equivalent to developing reliable methods for numerically solving a fully nonlinear partial differential equation, known as the Monge-Ampère equation. This project will investigate several questions regarding the approximation of solutions to Monge-Ampère equations. Of particular interest will be the numerical implementation of a dynamic approximation of optimal mappings using nonlinear parabolic equations of Monge-Ampère type, something that was investigated in previous works by the co-mentors.

Prerequisites: Some exposure to differential equations and/or real analysis.