Graph Theory: Algebraically Defined Edge-Colored Graphs

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A proper edge-coloring of a graph $G$ is an assignment of colors to the edges of the graph such that no two adjacent edges have the same color. The following graphs show one 10-cycle that is properly colored with 2 colors and one that is not. Classic questions of interest related to properly edge-colored graphs include the minimum number of colors needed to properly color a graph as well as the effect of edge coloring on properties of graphs.

In an algebraically defined bipartite graph, each vertex is assigned an $(x, y)$-coordinate pair and two vertices $(x_1, y_1)$ and $(x_2, y_2)$ are adjacent if a certain algebraic expression is satisfied. This project aims to study properties of edge-colored complete bipartite graphs obtained by assigning edge colors determined by various algebraic conditions. We will consider how different algebraic restrictions affect several graph properties.

Applicants should have completed a course in abstract algebra. Prior knowledge of topics in graph theory and some programming experience may be helpful, but are not necessary.