# Graph Theory: Algebraically Defined Edge-Colored Graphs 

Advisors: Brian G. Kronenthal and Karen B. McCready

A proper edge-coloring of a graph $G$ is an assignment of colors to the edges of the graph such that no two adjacent edges have the same color. The following graphs show one $10-$ cycle that is properly colored with 2 colors and one that is not. Classic questions of interest related to properly edge-colored graphs include the minimum number of colors needed to properly color a graph as well as the effect of edge coloring on properties of graphs.


Properly Colored


Not properly colored

In an algebraically defined bipartite graph, each vertex is assigned an $(x, y)$-coordinate pair and two vertices $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ are adjacent if a certain algebraic expression is satisfied. This project aims to study properties of edge-colored complete bipartite graphs obtained by assigning edge colors determined by various algebraic conditions. We will consider how different algebraic restrictions affect several graph properties.

Applicants should have completed a course in abstract algebra. Prior knowledge of topics in graph theory and some programming experience may be helpful, but are not necessary.

