Agent Based Models and Sensitivity Analysis – Professor Allie Lewis

Outline: Agent-based models (ABMs) are capable of simulating complex biological phenomena at the microscale level by tracking individual agents whose behavior may be governed by a variety of different mechanisms. However, this level of model complexity comes at a cost; ABMs are often computationally expensive, such that a single simulation may take hours or even days to run. Due to this cost, many researchers forgo conducting adequate sensitivity analyses on ABMs, even when such analyses are vital to understanding the robustness of the model.

Borgonovo et. al (2022) recently outlined a procedure for conducting a rigorous sensitivity analysis on ABMs. However, the proposed procedure may still be highly limited by computational budget unless we can create a more intelligent sampling plan, in the sense that we limit model evaluations to input regimes in which the quantity of interest is likely to be highly sensitive to fluctuations in the parameter values. One commonly employed method for global sensitivity analysis is Morris screening (Morris, 1991), in which inputs are ranked according to their sensitivity by averaging over coarse, local derivatives. As detailed in Lewis et. al (2016), the use of active subspace methodology to exploit linear combinations of parameters which are most influential to the quantity of interest may be combined with the Morris sensitivity metric in order to improve the sampling mechanism. By applying such a sampling strategy to the sensitivity analysis procedure for an agent-based model, we can make intelligent choices about how to spend our limited computational budget.

Research objectives: To enable more efficient sensitivity analyses of ABMs by aligning the sampling strategy according to the most influential directions in the input space. Computational cost and feasibility will be compared with previously employed methods to illustrate that the proposed sampling scheme can produce the same sensitivity results with fewer model evaluations.

Outcomes: Active subspaces are one of an emerging set of tools for dimension reduction in the field of uncertainty quantification (Constantine, 2015). Students on this project will explore the tradeoffs between model complexity and computational costs, and will learn how to apply techniques of dimension reduction to allow for robust model analyses. In addition to developing and enhancing skills in linear algebra and probability, students will gain extensive experience in writing simulation code.

Pre-requisites: Some experience with linear algebra and/or calculus-based probability recommended. Programming experience preferred.

References:

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Nonlinear Evolution Equations and Soliton Theory - Professor Lanre Akinyemi

Outline: The nonlinear evolution equations have played a prominent role in describing nonlinear scientific phenomena which include optical fibers, plasma physics, fluid mechanics, hydrodynamics and theory of turbulence, water waves, solitary waves theory, chaos theory, chemical physics, and other areas. One of the most important and fundamental tasks in applied sciences and engineering is the development of exact traveling waves or approximate solutions for nonlinear evolution equations.
A solitary wave or soliton is a self-reinforcing wave packet that maintains its structure while moving at a constant speed. Solitons play an important role in many mathematical physics models and have a wide range of uses in science and engineering. Soliton theory has been used in biology to explain low frequency collective motion in DNA and proteins, as well as how energy and signal travel through biomembranes and the nervous system. The study of plasmas, which are composed of many charged particles, also makes extensive use of soliton phenomena. For instance, Dusty plasmas, which are composed of minute charged dust particles, have been studied using nonlinear oscillator chains that allow for various solitary-wave solutions. A breather is a localized periodic solution of either continuous media equations or discrete lattice equations. Breathers are solitonic structures. On the other hand, a lump solution is a rational function solution which is real analytic and decays in all directions of space variables.

Research objectives: In this work, we will first check the integrability of some nonlinear evolution equations through Painleve analysis via WTC-Kruskal algorithm. Then, we construct the multi-solitons, breather solutions, lump waves, and some waves interaction utilizing different ansatz's functions based on bilinear formalism and symbolic computation (Mathematica and Matlab Package). We will use some analytical method to acquire bright soliton, dark soliton, singular soliton, and periodic solutions. Sometimes it is very difficult to find exact solutions of some nonlinear models via analytical methods, so in this case we will also study their respective numerical (approximation) solutions via homotopy techniques. The nonlinear evolution equations that will be study are listed below:

a. Higher-order (seventh-order) modified Korteweg-de Vries equation**.**
b. Ivancevic option pricingmodel with unstable or stochastic term.

The Korteweg-de Vries (KdV) equation describes weakly and nonlinearly interacting shallow water waves, ion acoustic waves in a plasma, long internal waves in a density-stratified ocean, and acoustic waves on a crystal lattice. The KdV equation is one of the most well-known models for solitons, and it serves as a solid foundation for the study of other equations. The Ivancevic option pricing model is an alternative of the standard Black–Scholes pricing equation, which signifies a controlled Brownian motion related to the nonlinear Schrodinger equation.

Outcomes: The students will play a central role in the entire process, making conjectures, and working through proofs in journal articles. They will learn how to use analytical and numerical techniques like modified sub-equation method, Hirota bilinear method, q-homotopy analysis transform method, and homotopy perturbation transform method (both uses combine form of homotopy and Laplace transform) to study the exact and approximate solutions of some nonlinear evolution equations as well as to demonstrate their wave pattern and interactions. We hope to gain insight into the behavior of the obtained solutions through graphs.

Prerequisites: Calculus I & II (Recommended), Differential equations (Optional), Some programming experience.

References:

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Orderings of Point Sets – Professor Carl Hammarsten

Outline: Consider a set *S* of *n* points in the plane. A choice of a vantage point *p* in the plane then defines an ordering *O* on *S* according to each point's distance from *p*. For $n > 3$, only a specific subset of the *n!* possible orderings are achievable in this way. Good and Tideman have shown the maximum number of attainable orderings is a sum of unsigned Sterling numbers of the first kind. Carbonero, et al, have shown the minimum number of attainable orderings is $2n – 2$. They have also constructed a complete list of possibilities for the $n = 4$ case and derived a partial list of restrictions for the lists of possibilities when $n > 4$. In yet-to-be-published work from last summer, another REU group derived the full list of possibilities for $n = 5$ and developed techniques that should ideally extend to higher values of n.

Research Objectives: We will endeavor to find complete descriptions of the achievable number of orders for $n > 5$. As part of this goal, we will examine the related, though somewhat more general, question arising from considering a set *S*, a vantage point *p*, and a desired ordering *O*, in which case we ask what is the minimal modification to a point in (or subset of) *S* required to make *O* achievable.

Outcomes: Everyone in this group will play a major role in all of our work and hopefully feel equal ownership from the beginning. This project has a nice balance of identifying theoretical requirements and balancing those with constructive examples. Approaching the list of the possible number of orderings from both angles requires both strong proof writing technique and the ability to create unexpected examples.
The nature of this project also leads to many chances to modify and generalize ideas to different settings – i.e. by changing the ambient space or the number of observers. Students will thus have ample opportunity to experience the ever-changing experience of "research".

Prerequisites: Some experience with linear algebra and/or discrete mathematics recommended.

References:

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