Spring 2019

Barge Problem 8

Martian Fractions. You're about to leave Mars for your summer vacation, let's say on Jupiter. You need fuel for your trip, and your space ship has the following (very strange) property. Suppose the fuel tank currently reads $r = \frac{a}{b} < 1$, written in lowest possible terms. Your spaceship has two buttons, (T) and (B), that do the following:

- If you push the (T) button, the fraction $\frac{a}{b}$ changes to $\frac{a+1}{b}$, and then the new fraction gets reduced.
- If you push the (B) button, the fraction $\frac{a}{b}$ changes to $\frac{a}{b+1}$, and then the new fraction gets reduced, as before.

For example, here is a sequence of steps that transforms $\frac{5}{8}$ to $\frac{2}{3}$.

$$\frac{5}{8} \to \text{(T)} \to \frac{6}{8} = \frac{3}{4} \to \text{(B)} \to \frac{3}{5} \to \text{(B)} \to \frac{3}{6} = \frac{1}{2} \to \text{(B)} \to \frac{1}{3} \to \text{(T)} \to \frac{2}{3}$$

1. Suppose your tank is currently $\frac{2}{9}$ full, and you need it to be $\frac{11}{15}$ full to reach Jupiter. Show how to transform $\frac{2}{9}$ to $\frac{11}{15}$ using the buttons (T) and (B).
2. Let r_1 and r_2 be two distinct rationals between 0 and 1. Is it always possible to find a sequence of steps using the buttons (T) and (B) that will transform r_1 to r_2? If so, describe such a procedure; if not, give a specific example where this fails.

Barge Prizes

First Prize	\$1000
Second Prize	\$750
Third Prize	\$500

1. Form a team with other Lafayette students. Each team must have 3, 4 or 5 members.
2. The weekly problem will be posted online https://math.lafayette.edu/teambarge/ and in the Math Dept. There will be 8 problems during the semester.
3. Get your solution to Gary Gordon by Saturday, May 4. You can either turn in a hard copy or send your solution by email to `gordong@lafayette.edu`.
4. Don't Quit! Keep turning in problems, even if you're not 100% sure of your solution.

Due Date: Saturday, May 4.