Spring 2019

Barge Problem 7

Back to Infinite Mars. After a week off, you find yourself back on Mars, with a new problem. The Phobots (from Problem 5) left you with a puzzle. There are stepping stones numbered 1, 2, 3, ... that go all the way around the planet, starting where you live with stone number 1. Let's assume two crazy things:

1. Assume Mars is large enough to accommodate an infinite number of stones.
2. Also assume that each stone is occupied by a Phobot (a native of the Martian moon Phobos). This means there are an infinite number of Phobots.

Here is the game the Phobots play: Suppose a Phobot is standing on stone number n. If n is even, the Phobot moves to stone number $n/2$. If n is odd, the Phobot moves to stone number $n+1$. They keep doing this until they reach stone number 1. For example, the Phobot initially standing on stone number 11 would jump to the following stones: $11 \to 12 \to 6 \to 3 \to 4 \to 2 \to 1$.

1. Let a_n be the number of steps it takes to reach stone number 1 when you start at stone number n. For example, $a_{11} = 6$ from the above example.
   (a) Find a_{2019}.
   (b) Being closer doesn't mean you're closer: Find all numbers k with $k < 2019$ but $a_k > a_{2019}$,
2. Let b_n be the number of times the integer n appears as a_m for some m. For instance, $b_5 = 5$ because $a_m = 5$ for $m = 5, 12, 14, 15$, and 32, and no other values. Then $b_1, b_2, b_3, \ldots$ is a famous sequence. Find it, with proof.

Barge Prizes

First Prize	\$750
Second Prize	\$600
Third Prize	\$450

1. Form a team with other Lafayette students. Each team must have 3, 4 or 5 members.
2. Solve the Problem of the Week with your team. The weekly problem will be posted on the department web page and and in the Math Dept. There will be 8 problems during the semester.
3. Get your solution to Gary Gordon by Saturday, April 20. You can either turn in a hard copy or send your solution by email to `gordong@lafayette.edu`.
4. Dont Quit! Keep turning in problems, even if you're not 100% sure of your solution.

Deadline: Saturday, April 20.