

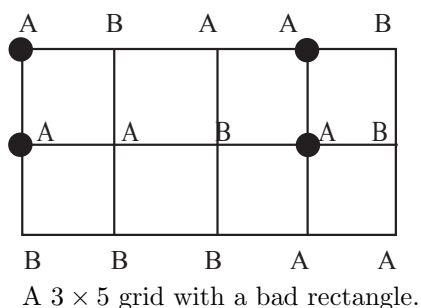
Spring 2019

Barge Problem 3

Gridlock. While living on Mars, you decide to plant potatoes at the corners of a $3 \times n$ grid. You have two kinds of potatoes, sweet and salty. (Botanists – avert your eyes.) You plant one kind of potato at each corner of the grid.

Because you have been living on Mars for a few months, by yourself, you're starting to go crazy. You decide that your grid cannot have 4 potatoes of the same kind planted at the corners of any rectangle in the entire grid. Call such a rectangle a *bad rectangle*. An example of a bad rectangle is given in the figure.

1. What is the largest value of n you can use so that your $3 \times n$ grid will have no bad rectangles? Find that value of n , show no larger value can work, or show that no such value exists.
2. Repeat part 1, but now with *squares* instead of rectangles.



Now suppose you decide to plant the potatoes in a $4 \times n$ grid.

3. Is there a smallest value of n that guarantees the existence of a bad rectangle? If so, find that value; if not, explain why not.
4. Same question, for squares.

This question has lots of **parts**, so there is lots of opportunity for **partial** credit.

Barge Prizes

First Prize	\$1000
Second Prize	\$750
Third Prize	\$500

1. Form a team with other Lafayette students. Each team must have 3, 4 or 5 members.
2. The weekly problem will be posted online <https://math.lafayette.edu/teambarge/> and in the Math Dept. There will be 8 problems during the semester.
3. Get your solution to Gary Gordon by Saturday, March 2. You can either turn in a hard copy or send your solution by email to gordong@lafayette.edu.
4. Don't Quit! Keep turning in problems, even if you're not 100% sure of your solution.

Due Date: Saturday, March 2.