Lafayette Problem Group
Problem Set 1

Try as many of these as you can by the next meeting, which will be **Thursday, September 18, at 4:10pm in Pardee 218 (Math Common Room) or a nearby room**. *Good Luck!.*

Problem 1: What's the value of $\sqrt{5 + \sqrt{5 + \sqrt{5 + \cdots}}}$?

Problem 2: Alice and Bob alternately roll a standard 6-sided die, with Alice going first. The winner is the person who first rolls the value that the other person just rolled (so Alice can't win on her first roll). What is the probability that Alice wins?

Problem 3: Five boys and five girls sit around a table. Does there have to be some person sitting directly between two girls? Why, or why not?

Problem 4: You may know that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$. (If you didn't, now you do.) Find a similar expression for the sum

$$1 + (1+2) + (1+2+3) + \cdots + (1+2+3+\cdots+n).$$

Problem 5: Of all isosceles triangles with a given perimeter, which have the greatest area?

Problem 6: A colony of chameleons currently contains 20 red, 18 blue, and 16 green individuals. When two chameleons of different colors meet, each of them changes his or her color to the third color. Is it possible that, after a while, all the chameleons have the same color? Why, or why not?

Problem 7[1]**:** Let A be any set of 20 distinct integers chosen from the arithmetic progression $1, 4, 7, \ldots, 100$. Prove that there must be two distinct integers in A whose sum is 104.

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[1]This problem comes from a former Putnam exam, a wickedly-difficult national math competition on the first Saturday of every December. This year's exam will take place from 10am until 6pm on **Saturday, December 6**. Don't worry if the problem is too hard - the median score on the Putnam exam is usually something like 1 out of 120. So you're never expected to do too well, which is a liberating concept. Just try it for fun!