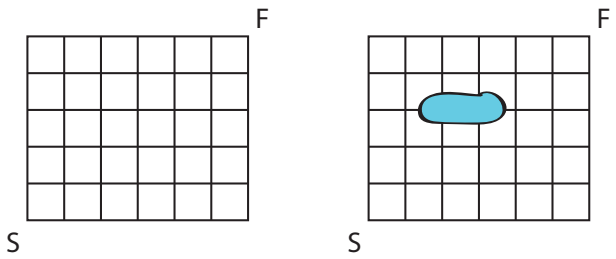
Lafayette Problem Group
Problem Set 2

Try as many of these as you can by the next meeting, which will be ***Thursday, September 19, from 4 to 5pm in Pardee 218 (Math Common Room)****. Good Luck!.*

Problem 1: Alice and Bob take turns flipping a fair coin, with Alice going first. The first player to flip the coin to land heads up wins. How much more likely is Alice to win than Bob?

Problem 2: How many paths are there from S to F in each of the grids below, if each step must either be to the north or to the east? For your information, that blob in the second grid is a lake filled with electric eels.



Problem 3: What are all of the possible positive integers a, b, c, and n such that

$$n^a + n^b = n^c \ ?$$

Problem 4: Suppose that all you know about a function $f : \mathbb{R} \to \mathbb{R}$ is that

- $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$
- $f(x) < 1000$ for all $x \in \mathbb{R}$

What's $f(1)$?

Turn over . . .

Problem 5: Any four points p_1, p_2, p_3, p_4 in the plane create six distances between pairs of points. Can you find four points in the plane so that all six distances are integers and so that no three points lie on the same line?

Problem 6[1]**:** A basketball player shoots some free throws and is successful on less than 80% of them. She keeps shooting more free throws and eventually is successful on more than 80% of all of them, including those in the first batch. Must there have been a moment when she was successful on *exactly* 80% of them?

[1]This problem is based on a former Putnam exam, a wickedly-difficult national math competition on the first Saturday of every December. This year's exam will take place from 10am until 6pm on **Saturday, December 7**. Don't worry if the problem is too hard - the median score on the Putnam exam is usually something like 1 out of 120. So you're never expected to do too well, which is a liberating concept. Just do it for fun!