Lafayette Problem Group
Problem Set 2

Try as many of these as you can by the next meeting, which will be ***Thursday, September 25, at 4:10pm in Pardee 218 (Math Common Room) or a nearby room****. Good Luck!.*

Problem 1: There are 100 coins on a table. For each of the numbers $1, 2, 3, \ldots, 20$ in order, you must turn over exactly i coins. Can you guarantee that at the end of this process the coins are either all face up or all face down?

Problem 2: Find all positive integers k satisfying $4 < \frac{k}{4} + \frac{4}{k} < 4.1$.

Problem 3: What's the probability of being dealt a five-card poker hand that's a full house? What's the probability of having a full house among the seven cards you see at the end of a round of Texas Hold 'Em? Look up "poker," "full house," and "Texas Hold 'Em" if you're not sure what these terms mean!

Problem 4[1]**:** Do there exist polynomials $a(x), b(x), c(y), d(y)$ such that the following holds identically?

$$1 + xy + x^2y^2 = a(x)c(y) + b(x)d(y)$$

.

Problem 5: *Here's Problem 1947 from* Mathematics Magazine*. If we solve this problem by November 1, we can submit our answer for publication.*
Let n be a positive integer. Prove that

$$\sum_{k=0}^{n} |\cos(k)| \geq \frac{n}{2}.$$

[1] This problem comes from a former Putnam exam, a wickedly-difficult national math competition on the first Saturday of every December. This year's exam will take place from 10am until 6pm on **Saturday, December 6**. Don't worry if the problem is too hard - the median score on the Putnam exam is usually something like 1 out of 120. So you're never expected to do too well, which is a liberating concept. Just try it for fun!