

Lafayette Problem Group

Problem Set 3

Try as many of these as you can by the next meeting, which will be Thursday, September 26, from 4 to 5pm in Pardee 218 (Math Common Room). Good Luck!

Problem 1: If you have an unlimited supply of 7 cent stamps and 10 cent stamps, what is the largest value of postage that you *can't* make with the stamps?

Problem 2: In how many different ways can the letters of BEEKEEPER be rearranged?

Problem 3: What is the maximum number of regions that n lines can divide the plane into? For example, the maximum number of regions is 7 when $n = 3$.

Problem 4: Suppose that all you know about a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is that $f(x + 1) = f(x) + 1$ for all $x \in \mathbb{R}$. Must the graph of f be a line?

Problem 5: 10 coins of various denominations are lined up in a row on a table. Alice and Bob play a game in which they alternate turns, with Alice going first. On each turn, the player must take a coin from either end of the line of coins that remain. Can Alice always play in such a way that she ends up with at least as much money as Bob once all of the coins have been removed from the table?

Problem 6¹: A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?

¹This problem is based on a former Putnam exam, a wickedly-difficult national math competition on the first Saturday of every December. This year's exam will take place from 10am until 6pm on **Saturday, December 7**. Don't worry if the problem is too hard - the median score on the Putnam exam is usually something like 1 out of 120. So you're never expected to do too well, which is a liberating concept. Just do it for fun!

And the problem we still can't solve ... Any four points p_1, p_2, p_3, p_4 in the plane create six distances between pairs of points. Can you find four points in the plane so that all six distances are different integers and so that no three points lie on the same line?