

Lafayette Problem Group

Problem Set 1

Try as many of these as you can by the next meeting, which will be Thursday, September 12, from 4 to 5pm in Pardee 218 (Math Common Room). Good Luck!.

Problem 1: How many positive integers less than 100 are multiples of either 3 or 4 but not multiples of 6?

Problem 2: Five boys and five girls sit around a table. Does there have to be some person sitting directly between two girls? Why, or why not?

Problem 3: Any four points p_1, p_2, p_3, p_4 in the plane create six distances between pairs of points. Can you find four points in the plane so that all six distances are integers? Can you do it if the four points can't lie on the same line? Is it possible to have the six integers be different?

Problem 4 (from Sam Loyd): “The top 'o the mornin' to you, officer,” said Mr. McGuire. “Can you tell me what time it is?”

“I can do the same,” replied officer Clancy. “Just add one quarter of the time from midnight until now to half the time from now until midnight, and it will give you the correct time.”

When did this conversation take place?

Problem 5: A colony of chameleons currently contains 20 red, 18 blue, and 16 green individuals. When two chameleons of different colors meet, each of them changes his or her color to the third color. Is it possible that, after a while, all the chameleons have the same color? Why, or why not?

Problem 6¹: Find, with explanation, the maximum value of $f(x) = x^3 - 3x$ on the set of all real numbers satisfying $x^4 + 36 \leq 13x^2$.

¹This problem comes from a former Putnam exam, a wickedly-difficult national math competition on the first Saturday of every December. This year's exam will take place from 10am until 6pm on **Saturday, December 7**. Don't worry if the problem is too hard - the median score on the Putnam exam is usually something like 1 out of 120. So you're never expected to do too well, which is a liberating concept. Just do it for fun!