

Lafayette Problem Group – Problem Set 6

Everyone is welcome! Try to get solutions, or good ideas, or even just bad ideas, for some of these problems by next week's meeting:

Thursday, October 23
Lunchtime in Pardee 216

Problem 1: Let $A_n = \{1, 2, \dots, n\}$. A subset B_n of A_n is called an *alternating subset* if when we list the elements of B_n in ascending order, B_n has the format { odd, even, odd, even, \dots , odd, even } or the format { even, odd, even, odd, \dots , even, odd }. (We assume that the empty set is an alternating subset of any A_n .)

Let $f(n)$ be the number of alternating subsets of A_n . Find $f(n)$.

Problem 2: How many different sequences with length n satisfy the following conditions: (1) The sequence contains 0 and 1 only. (2) 010 and 101 do not appear anywhere in the sequence. For example, when $n = 4$ there are 10 different sequences: 0000, 0001, 0011, 0110, 0111, 1111, 1110, 1100, 1001, and 1000.

Problem 3: The graph of a function $f(x)$ on \mathbb{R} is symmetric about the point $(1, 3)$ and the line $x = 4$. Show that $f(x)$ is a periodic function on \mathbb{R} .

Problem 4: Let $f(x)$ be a function on \mathbb{R} such that $f^{(n)}(x)$ exists everywhere on \mathbb{R} . Suppose that for any $h > 0$ we have

$$\frac{f(x+h) - f(x)}{h} = f'(x + \frac{h}{2}).$$

Find $f(x)$.

Problem 5: What values of a make the equation $\log_a(x) = x$ have a solution?

Problem 6: Assume that a_1, a_2, \dots, a_n is a permutation of $1, 2, \dots, n$. If n is odd, is $P = (a_1 - 1)(a_2 - 2) \dots (a_n - n)$ even, odd, or undetermined?

Remember to visit www.lafayette.edu/~math!