

## Lafayette Problem Group – Problem Set 3

*Everyone is welcome! Try to get solutions, or good ideas, or even just bad ideas, for some of these problems by next week's meeting:*

**Thursday, September 18**  
**Lunchtime in Pardee 216**

**Problem 1:** You are walking around in the  $xy$ -plane. You start at the origin, then move 1 unit east (to the point  $(1,0)$ ), then 2 steps north (to the point  $(1,2)$ ), then 3 steps west (to the point  $(-2,2)$ ), then 4 steps south, then 5 steps east, and so on. Where will you be after taking precisely 2008 steps?

**Problem 2:** You are playing a card game (like Texas Hold 'Em) where two cards are dealt to each player face down. You are interested in the probability that the opponent to your right was just dealt two aces, so you compute that value before knowing anything about your own cards.

Before you have a chance to look at your cards, you catch just a glimpse of one of your opponent's cards; all you can conclude is that it is a heart. Does that change the probability that your opponent has two aces?

**Problem 3:** Here's a number thing:  $1 \times 2 \times 3 \times 4 = 24 = 5^2 - 1$ . Show that the product of 4 consecutive positive integers is always one less than a perfect square.

**Problem 4:** You roll an ordinary die (a cube with the numbers 1 - 6) repeatedly and keep track of the running sum. You stop rolling as soon as this running sum is greater than 24. What is the most likely value of the sum when you stop?

**Problem 5:** *The LVAIC Exam will take place this fall, and you can participate! More details will be forthcoming, but for now, here's a problem from last year's exam.*

Find the last digit of  $2007^{(2007^{2007})}$ .

**Problem 6:** *Here's a problem from a recent Putnam exam.*

Show that if the points of an isosceles right triangle of side length 1 are each colored with one of four colors, then there must be two points of the same color which are at least a distance  $2 - \sqrt{2}$  apart.

**Remember to visit [www.lafayette.edu/~math/](http://www.lafayette.edu/~math/)!**