Math 3XX: Lattices, and More Lattices!

A Special Topics course for Fall 2006, possibly MWF at 11:00
Professor Derek Smith (Pardee 224)

There are two common types of “lattices” in mathematics. Here is an example of each kind:

The figure on the left is a certain type of partially-ordered set, or “poset,” pronounced “POH-set.” It describes the relationships among six elements, in this case colleges and universities – can you determine the rule for the given ordering? A lattice is a certain type of poset. Applications of lattices and posets abound, particularly in computer science and the social sciences.

The figure on the right is only a small portion of another type of lattice. The entire lattice would extend infinitely in all directions, with a point drawn for each point \((x, y)\) in the plane \(\mathbb{R}^2\) for which both \(x\) and \(y\) are integers. You can think of this lattice as being “tiled” by squares if you wish; a different lattice in the plane would be one tiled by equilateral triangles – try drawing that now! These types of lattices, especially those in three and even higher dimensions, find applications in many areas, from cryptography (code-breaking) to crystallography (in chemistry).

Think of this course as two introductory half-courses, one on lattices and the other on lattices! A focus in each half of the course will be determining and then proving “classification theorems” for certain important classes of lattices, with applications of lattice theory to other branches of mathematics (eg. number theory) and science discussed along the way.

Prerequisite: Math 275, or permission of instructor.